Math 120: Final Exam Practice

These problems are intended to help you in your studying for your final exam. With each problem, I have also included a reference in your text for additional practice on the related topics. Please ask me if you have any questions or need clarification on any of these problems. This set of problems is (approximately) of the same length and difficulty level of a final exam. Inclusion (or exclusion) of a particular technique or concept from these problems does not imply inclusion (or exclusion) of said technique on the exam.

1. Compute the derivative of each of the following functions. Please show your work. (see also: Chapter 3 review, exercises 1-50)

   (a) $f(t) = \cot(3t^2 + 5)$

   (b) $g(x) = \frac{\sqrt{5x-16}(x+2)}{x^2-1}$

   (c) $h(x) = 5^{x^2+3x}$

   (d) $f(x) = x^{x^x}$

2. Calculate each of the following limits. Please show your work. (see also: Chapter 2 review, Exercises 3-20, Chapter 4 review, Exercises 7-14)

   (a) $\lim_{x\to 2} \dfrac{x - \sqrt{2+x}}{x^2 - 2x}$

   (b) $\lim_{x\to\infty} \dfrac{\sin x}{x+3}$

   (c) $\lim_{x\to\infty} \sqrt{x}e^{-x^2}$

   (d) $\lim_{x\to 1^+} x^{1/(1-x)}$

3. A local sports team plays in an arena with seating for 20,000 people. When ticket prices are set at \$90, average attendance is 14,000. A market research firm determines that for every \$5 the ticket prices are lowered, an average of 1,000 more people attend games. Determine the price the team should set for tickets so that the revenue from ticket sales is optimal. (see also: Section 4.7, Chapter 4 review, Exercises 50-59)

4. The volume of a cube is increasing at a rate of 10 cm^3/hr. How fast is the surface area increasing when the length of an edge is 30 cm? (see also: Chapter 4 review, Exercises 90-100)

5. Show that the equation $3x + 2\cos(x) + 5 = 0$ has exactly one real solution. (see also: Section 4.2)

6. Sketch the graph of a function f that is continuous on $(-5, 5)$, with $f(0) = 1$, $f'(0) = 1$, $f'(-2) = 0$, $\lim_{x\to -5^+} f(x) = \infty$, and $\lim_{x\to 5^-} = 3$. (see also: Sections 2.7, 2.8)

7. Compute each of the following integrals. Please show your work. (see also: Chapter 4 review, Exercises 65-72, Chapter 5 review, Exercises 11-40, Chapter 7 review, Exercises 1-7, 9-12, 20, 24, 26, 36, 38)

   (a) $\displaystyle\int xe^x \sin x \, dx$

   (b) $\displaystyle\int t^3\sqrt{t^2+1} \, dt$

   (c) $\displaystyle\int \tan^5\theta \sec^3\theta \, d\theta.$

   (d) $\displaystyle\int_{\pi/2}^{\pi} (\sin^2 x + 2)\cos x \, dx$

8. Let $f(x) = x^3 - x^2$.

   (a) Write a Riemann sum for approximating $\int_1^3 f(x)\,dx$ having four subintervals.

   (b) Draw a picture showing what your approximation from part (a) represents.

   (c) Write the value of $\int_1^3 f(x)\,dx$ as a limit of Riemann sums. You need not calculate the limit.

   (d) Calculate the actual value of $\int_1^3 f(x)\,dx$.

   (see also: Section 5.1, problems 1-8, Section 5.2, problems 1-6, 17-20, 26-30)

9. The region enclosed by the curves $y = x, y = 0, x = 2, x = 4$ is rotated around the axis $x = 1$. Determine the volume of the resulting solid. (see also: Section 6.2, problems 1-30)

10. Mark each of the following items as true or false. Provide a short (1-2 sentences) explanation for your response. (see also: T/F in chapter review for chapters 2, 3, 4, 5)

    (a) If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$.

    (b) If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then $\lim_{x\to a}(f(x) - g(x)) = 0$.

    (c) If $f(x)$ does not have a horizontal asymptote, then $\lim_{x\to\infty} f(x) = \pm\infty$.

    (d) If $y = x^x$, then $y' = x \cdot x^{x-1}$.

    (e) If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

    (f) If $\int_0^x f(t)dt = \int_0^x g(t)dt$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

    (g) Every differentiable function is integrable.

    (h) Every integrable function is differentiable.

    (i) If f and g are differentiable and $f(x) \geq g(x)$ for $0 < x < 1$, then $f'(x) \geq g'(x)$ for $0 < x < 1$.

    (j) If f and g are integrable and $f(x) \geq g(x)$ for $0 < x < 1$, then $\int_0^x f(t)\,dt \geq \int_0^x g(t)\,dt$ for $0 < x < 1$.

11. Answer each of the following questions. (see also: concept check in chapter review for chapters 2, 3, 4, 5)

    (a) What is the Squeeze Theorem? Explain why it works.

    (b) Write the definition of a derivative. Explain it in terms of slopes.

    (c) Explain the difference between a global maximum and a local maximum.

    (d) What does the Fundamental Theorem of Calculus say? (you may state either part) Why is it important?

    (e) Write the integral $\int_a^b f(u(x))u'(x)\,dx$ as a definite integral with respect to u.

Some other topics, not included on this practice exam:

1. Exponential growth/decay: Section 3.8

2. Linear approximations: Section 3.10

3. Hyperbolic trig: Section 3.11
4. Graph sketching: Sections 4.3, 4.5
5. Integral as net change: Section 5.4
6. Area between curves: Section 6.1