Math 101 Homework

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Complete the following problems. Fully justify each response.

1. State the Axiom of Choice (in your own words). Explain at least one circumstance where we can select elements from a family of sets that does NOT require Choice.
2. Let $X = \{(a, b) \mid a, b \geq 0\}$, the first quadrant. Define $\leq$ by $(a, b) \leq (c, d)$ whenever $ac > bd$. Is this a partial order? (Hint: look at transitivity)
3. Let $X = \{(a, b) \mid a, b \geq 0\}$, the first quadrant. Define $\leq$ as follows:
   (a) If $a < c$, then $(a, b) < (c, d)$.
   (b) If $a = c$ and $b < d$, then $(a, b) < (c, d)$.

   Show that this is a partial order on X. (This is a common ordering, called lexicographical ordering, or dictionary ordering.)
4. State the De Bruijn-Erdős Theorem, and explain its relevance to the Hadwiger-Nelson problem.