A model theoretic solution to a problem of László Fuchs

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Problem 5.1 in page 181 of *Abelian Groups* by László Fuchs, asks to find the cardinals $\lambda$ such that there is a universal abelian $p$-group for purity of cardinality $\lambda$, i.e., an abelian $p$-group $U$ of cardinality $\lambda$ such that every abelian $p$-group of cardinality $\lambda$ purely embeds in $U$. In this series of talks, we will use ideas from the theory of abstract elementary classes to show that there are many cardinals where there is a universal abelian $p$-group for purity. Moreover, we will provide a complete solution to Fuchs' question below $\aleph_\omega$ except for $\aleph_0$ and $\aleph_1$. We will introduce all the notions of abstract elementary classes needed to follow the argument of the main theorem and give plenty of algebraic examples to showcase them. The lectures are based on the following paper “A model theoretic solution to a problem of László Fuchs”.