Practice Math Contest | Misha Lavrov

The Largest Prime Factor Function

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In the problems[1] below, let $P(n)$ denote the largest prime factor of n. For example, since $2016 = 2^5 \cdot 3^2 \cdot 7$, $P(2016) = 7$; since 2017 is prime, $P(2017) = 2017$.

1. (a) Find $P(100! + 101!)$.

   (b) Find the largest *2-digit* prime factor of $\binom{200}{100}$.

2. Prove that there are infinitely many integers n such that $P(n) < P(n+1) < P(n+2)$.

3. Prove that there are infinitely many triples of distinct positive integers (a, b, c) such that $P(a^2+1) = P(b^2+1) = P(c^2+1)$.

[1] Problems 1(a) and 1(b) are taken from posts on the Art of Problem Solving forum, with slight modification. Problems 2 and 3 are taken from posts on `http://www.reddit.com/r/mathriddles/`.