In the problems below, let $P(n)$ denote the largest prime factor of $n$. For example, since $2016 = 2^5 \cdot 3^2 \cdot 7$, $P(2016) = 7$; since 2017 is prime, $P(2017) = 2017$.

1. (a) Find $P(100! + 101!)$.

(b) Find the largest 2-digit prime factor of $\binom{200}{100}$.

2. Prove that there are infinitely many integers $n$ such that $P(n) < P(n + 1) < P(n + 2)$.

3. Prove that there are infinitely many triples of distinct positive integers $(a, b, c)$ such that $P(a^2 + 1) = P(b^2 + 1) = P(c^2 + 1)$.

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1 Problems 1(a) and 1(b) are taken from posts on the Art of Problem Solving forum, with slight modification. Problems 2 and 3 are taken from posts on [http://www.reddit.com/r/mathriddles/](http://www.reddit.com/r/mathriddles/).