MATH 21123 Quiz 5 02/24/11

Name: Solution key Lab Section: _____

Allowed time: 12 mins

1. (3 points) Give the value of a_n for the following functions. (HINT: Use your memory!)

   (a) $e^x = \sum_{n=0}^{\infty} a_n x^n$.

   $= \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\quad \boxed{a_n = \frac{1}{n!}}$

   (b) $\sin x = \sum_{n=0}^{\infty} a_n x^n$.

   $= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $\quad a_n = \begin{cases} (-1)^k \frac{1}{(2k+1)!} & \text{when } n = 2k+1 \\ 0 & \text{when } n = 2k \end{cases}$

   (c) $\cos x = \sum_{n=0}^{\infty} a_n x^n$.

   $= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $\quad a_n = \begin{cases} (-1)^k \frac{1}{(2k)!} & \text{when } n = 2k \\ 0 & \text{when } n = 2k+1 \end{cases}$

2. (2 points) Use one of the answer from Question no. 1 to find the sum of the series $\sum_{n=0}^{\infty} (-1)^n \frac{(\ln 2)^n}{n!}$.

   $= \sum_{0}^{\infty} \frac{1}{n!} (-\ln 2)^n = e^{-\ln 2}$ (part (a))

   $= \frac{1}{2}$ Ans

3. (5 points) Give the Taylor polynomial of $f(x) = 1/x$ of degree two centered at $c = 1$.

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$\frac{1}{x}$	1
1	$-\frac{1}{x^2}$	-1
2	$\frac{2}{x^3}$	$2 = 2!$
3	$-\frac{3(2)}{x^4}$	$-3(2) = -3!$
4	$\frac{4(3)(2)}{x^5}$	$4(3)(2) = 4!$
$\vdots$	$\vdots$	$\vdots$

DON'T NEED!

$T_2(1) = 1 - (x-1) + \frac{2(x-1)^2}{2!}$

$= 1 - (x-1) + (x-1)^2$ Ans