MATH 21123 Quiz 4 02/17/11

Name: Solution key Lab Section: ____________

Allowed time: 15 mins

1. (4 points) Find the radius of the convergence of the power series $\sum_{k=1}^{\infty}(-1)^k\frac{(x+2)^k}{k2^k}$. Also, find the interval of convergence.

$$a_k = (-1)^k \frac{(x+2)^k}{k\,2^k}$$

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{(x+2)}{2}\,\frac{k}{k+1}\right| = \frac{|x+2|}{2} < 1$$

$\Rightarrow$ R.O.C $= 2$ Ans and $-4 < x < 0$

At end points: $x=-4$ $\quad \sum_1^{\infty}\frac{(-1)^k(x-2)^k}{k\,2^k} = \sum_1^{\infty}\frac{1}{k}$ diverges

$x = 0$ $\quad \sum_1^{\infty}(-1)^k\frac{2^k}{k\,2^k} = \sum\frac{(-1)^n}{k}$ converges.

I.O.C $= (-4, 0]$ Ans

2. (6 points) Let $f(x) = \frac{1}{x+2}$. Express $f,\ f',\int f(x)\,dx$ as a power series centered at $x=0$, that is, express it as $\sum a_n x^n$.

$$f(x) = \frac{1}{2\left(\frac{x}{2}+1\right)} = \frac{1}{2}\sum_0^{\infty}\left(\frac{-x}{2}\right)^n$$

$$f'(x) = \frac{1}{2}\sum_1^{\infty}(-1)^n\frac{n\,x^n}{2^n}$$

$$\int f(x)\,dx = \frac{1}{2}\sum_0^{\infty}\frac{(-1)^n}{2^n}\frac{x^{n+1}}{n+1} + C$$

To find C note that L.H.S $= \int\frac{1}{x+2}\,dx = \ln|x+2|$

Plug in $x=0$ $\Rightarrow$ $C = \ln 2$

$$\int f(x)\,dx = \frac{1}{2}\sum_0^{\infty}\frac{(-1)^n}{2^n}\frac{x^{n+1}}{n+1} + \ln 2$$

Ans