MATH 21123 Quiz 3 02/03/11

Name: Solution key Lab Section: ______________

Allowed time: 15 mins

1. (4 points) Determine whether the series $\sum(-1)^k(\sqrt{k+1}+\sqrt{k})^k$ is absolutely convergent, conditionally convergent or divergent. Show all your work.

$$a_k = (-1)^k\left(\sqrt{k+1}+\sqrt{k}\right)^k$$

$$\lim_{k\to\infty} |a_k| = \infty \Rightarrow \lim_{k\to\infty} a_k \text{ does not exist.}$$

$$\Rightarrow \sum(-1)^k\left(\sqrt{k+1}+\sqrt{k}\right)^k \text{ diverges by divergence test.}$$

2. (4 points) Determine whether the series $\sum(-1)^k\frac{k!}{k^k}$ is absolutely convergent, conditionally convergent or divergent. Show all your work.

$$a_k = (-1)^k \frac{k!}{k^k}$$

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \frac{\cancel{(k+1)}}{(k+1)^{\cancel{k+1}}}\cdot k^k = \lim_{k\to\infty}\left(\frac{k}{k+1}\right)^k = \lim_{k\to\infty}\frac{1}{\left(1+\frac{1}{k}\right)^k} = \frac{1}{e}$$

Note that $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| < 1 \Rightarrow \sum_k a_k$ is absolutely convergent.

3. (2 points) Using Question 2, give the value of $\lim_{k\to\infty}\frac{k!}{k^k}$.

Note $\sum a_k$ converges absolutely $\Rightarrow \lim_{k\to\infty}|a_k| = 0$

$$\Rightarrow \boxed{\lim_{k\to\infty}\frac{k!}{k^k} = 0}$$