MATH 21123 Quiz 2 1/21/11

Name: Key Lab Section: _____

1. Determine whether the series $\sum \frac{1}{n^2-1}$ converge or diverge. Find the limit if it converges. (3) points

$$a_n = \frac{1}{n^2-1} = \frac{1}{2}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

Note $n=2 \longrightarrow$

$$S_n = \frac{1}{2}\left[1 - \frac{1}{3}\right] + \frac{1}{2}\left[\frac{1}{2} - \frac{1}{4}\right] + \cdots + \frac{1}{2}\left[\frac{1}{n-2} - \frac{1}{n}\right] + \frac{1}{2}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$S_n = \frac{1}{2} + \frac{1}{4} - \frac{1}{2n} - \frac{1}{2(n+1)}, \quad \lim_{n\to\infty} S_n = \frac{1}{2} + \frac{1}{4} = \frac{6}{8} \text{ Ans}$$

2. Does there exist a value of r so that the geometric series $\sum_{n=1}^{\infty} r^n$ converges and has sum equal to 1? If yes, then give the value of r. (2) points

Know $\sum_{n=1}^{\infty} r^n$ converges iff $|r|<1$ and $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$

$$\Rightarrow \frac{r}{1-r} = 1 \text{ and } |r|<1 \Rightarrow r = \frac{1}{2} < 1$$

Yes, there exist such a value and it is equal to $\frac{1}{2}$.

3. Does there exist a value of s so that the geometric series $\sum_{n=0}^{\infty} s^n$ converges and has sum equal to $\frac{1}{3}$? If yes, then give the value of s. (2) points

Know: $\sum s^n$ converges iff $|s|<$ and $\sum_{n=0}^{\infty} s^n = \frac{1}{1-s}$

$$\Leftrightarrow \frac{1}{1-s} = \frac{1}{3} \text{ and } |s|<1 \Leftrightarrow s = -2 \text{ and } |s|<1$$

which is not true. There does not exist a value of s so that the sum $= \frac{1}{3}$.

4. If the fourth term in geometric series $\sum_{n=0}^{\infty} ar^n$ is $\frac{4}{3}$ and the seventh term is $\frac{32}{81}$ then find the value of the common ratio. (3) points

$$ar^4 = \frac{4}{3}$$

$$ar^7 = \frac{32}{81} \Rightarrow ar^4 \cdot r^3 = \frac{32}{81}$$

$$\Rightarrow \frac{4}{3} r^3 = \frac{32}{81} \Rightarrow r^3 = \frac{8}{27}$$

$$\Rightarrow \boxed{r = \frac{2}{3}} \text{ Ans}$$