21-228 Discrete Mathematics

Assignment 1

Due Fri Jan 27, at start of class

Notes: Collaboration is permitted except in the writing stage. Also, please justify every numerical answer with an explanation.

1. Recall that $A \triangle B$ denotes the symmetric difference of A and B, i.e., the set of all x that belong to exactly one of A or B. Simplify:

$$\big((A \triangle B) \triangle (B \triangle C)\big) \triangle (C \triangle A).$$

   If you wish, you may express your answer in the form of a Venn diagram, with the final set shaded in.

2. Let $n \geq 3$. How many subsets of $\{1, 2, \ldots, n\}$ contain exactly two of the integers 1, 2, 3? For example, $\{1, 2, 7, 9\}$ and $\{1, 3, 9\}$ would count, but $\{1, 7, 9\}$ would not.

3. King Kong has escaped, and is at the southwest corner of Central Park (59th St / 8th Ave). He wants to get to the Empire State Building (34th St / 5th Ave) as quickly as possible, but he must avoid Times Square (42nd St / 7th Ave). If he always takes the most direct route, in how many ways can this be done? Assume the streets form a perfect grid, i.e., ignore Broadway, parks, etc. Answers may be expressed in terms of factorials or binomial coefficients, but summation notation and ellipses may not be used.

4. ($\star$) A 4-digit number is called a *palindrome* if it is the same when the digits are read in reverse. For example, 7337 and 3333 are 4-digit palindromes, but 1337 and 0990 are not. Note that 0990 doesn't count because it's actually a 3-digit number.

   A 4-digit number is called an *almost-palindrome* if there is a way to change exactly one digit so that the result is a 4-digit palindrome. For example, 1337, 1501, and 1990 are 4-digit almost-palindromes (they could become 1331 or 7337, 1001 or 1551, and 1991), but 1234, 0991, and 1331 are not. The issue with 0991 is again that it is actually a 3-digit number, and the issue with 1331 is that if you change any digit, then it becomes a non-palindrome.

   How many 4-digit almost-palindromes are there?