Extremal Combinatorics: Homework 2

Due Friday, April 8, in class

1. First, convince yourself that every tree has chromatic number $\chi \leq 2$. (You don't need to write that first part up.) Next, show that every graph which is the superposition of two possibly-overlapping trees on the same vertex set has chromatic number $\chi \leq 4$. Then show that every superposition of three possibly-overlapping trees on the same vertex set has chromatic number $\chi \leq 8$. Finally, determine the minimum integer C such that every superposition of three possibly-overlapping trees on the same vertex set has chromatic number $\chi \leq C$.

2. For some infinite set of values of t, prove the following statement: every n-vertex graph with average degree d contains at least nd^t distinct walks of length t. These are sequences of $t+1$ (possibly repeated) vertices $v_0, \ldots, v_t$, where each $v_i v_{i+1}$ is an edge. Note that you should prove this result not up to a constant factor: you should get a bound of really at least nd^t.

3. Is it true that for every integer n, there is an n-vertex graph with at least $10^{-6}n^2$ edges, each of which is contained in at least one triangle, but with no edge in more than 10^6 triangles?