

Extremal Combinatorics: Homework 1

Due Wednesday March 2, in class

1. It is known that for every assignment of a distinct real number to each edge of K_n , there is a simple path (not reusing vertices) of length at least about \sqrt{n} , on which the edge labels increase monotonically. Try this yourself! You'll get full credit for proving any bound which tends to infinity. For example, if you prove that there is always an increasing simple path of length at least $\log n$, this will count.
2. Let A and B be two disjoint sets of n vertices each. Let M_1, M_2, \dots, M_n be edge-disjoint matchings from A to B , and let U_1, U_2, \dots, U_n be their corresponding vertex sets. Let G be the graph with vertex set $A \cup B$ and edge set equal to the union of all the edges of the matchings.

We say that this is a collection of induced matchings if each M_i is exactly the induced subgraph $G[U_i]$. A famous result says that then G has only $o(n^2)$ edges. Instead of assuming this "induced" condition, let's instead assume that for every matching M_i , every pair of its edges $\{a_1b_1, a_2b_2\}$ (with $a_j \in A$ and $b_j \in B$) has the property that there is no $b' \in B$ where both a_1b' and a_2b' are in G . Note that this is a stronger condition than the induced matching condition. Show that under this condition, G has only $O(n^{3/2})$ edges.

3. Let G be an n -vertex graph with strictly more than $n(t - 2)/2$ edges. Show that it must contain a simple path (not reusing vertices) with at least t vertices.