# Putnam Σ.14

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## 1 Problems

**Putnam 1998/A4.** Let $A_1 = 0$ and $A_2 = 1$. For $n > 2$, the number $A_n$ is defined by concatenating the decimal expansions of $A_{n-1}$ and $A_{n-2}$ from left to right. For example $A_3 = A_2A_1 = 10$, $A_4 = A_3A_2 = 101$, $A_5 = A_4A_3 = 10110$, and so forth. Determine all $n$ such that 11 divides $A_n$.

**Putnam 1998/A5.** Let $\mathcal{F}$ be a finite collection of open discs in $\mathbb{R}^2$ whose union contains a set $E \subseteq \mathbb{R}^2$. Show that there is a pairwise disjoint subcollection $D_1, \ldots, D_n$ in $\mathcal{F}$ such that

$$E \subseteq \cup_{j=1}^{n} 3D_j.$$

Here, if $D$ is the disc of radius $r$ and center $P$, then $3D$ is the disc of radius $3r$ and center $P$.

**Putnam 1998/A6.** Let $A, B, C$ denote distinct points with integer coordinates in $\mathbb{R}^2$. Prove that if

$$(|AB| + |BC|)^2 < 8 \cdot [ABC] + 1$$

then $A, B, C$ are three vertices of a square. Here $|XY|$ is the length of segment $XY$ and $[ABC]$ is the area of triangle $ABC$.