

# Putnam S.10

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## 1 Problems

**Putnam 2000/B4.** Let  $f(x)$  be a continuous function such that  $f(2x^2 - 1) = 2xf(x)$  for all  $x$ . Show that  $f(x) = 0$  for  $-1 \leq x \leq 1$ .

**Putnam 2000/B5.** Let  $S_0$  be a finite set of positive integers. We define finite sets  $S_1, S_2, \dots$  of positive integers as follows: the integer  $a$  is in  $S_{n+1}$  if and only if exactly one of  $a - 1$  or  $a$  is in  $S_n$ . Show that there exist infinitely many integers  $N$  for which  $S_N = S_0 \cup \{N + a : a \in S_0\}$ .

**Putnam 2000/B6.** Let  $B$  be a set of more than  $2^{n+1}/n$  distinct points with coordinates of the form  $(\pm 1, \pm 1, \dots, \pm 1)$  in  $n$ -dimensional space with  $n \geq 3$ . Show that there are three distinct points in  $B$  which are the vertices of an equilateral triangle.