

Putnam S.8

Po-Shen Loh

18 October 2015

1 Problems

Putnam 2000/A4. Show that the improper integral

$$\lim_{B \rightarrow \infty} \int_0^B \sin(x) \sin(x^2) dx$$

converges.

Putnam 2000/A5. Three distinct points with integer coordinates lie in the plane on a circle of radius $r > 0$. Show that two of these points are separated by a distance of at least $r^{1/3}$.

Putnam 2000/A6. Let $f(x)$ be a polynomial with integer coefficients. Define a sequence a_0, a_1, \dots of integers such that $a_0 = 0$ and $a_{n+1} = f(a_n)$ for all $n \geq 0$. Prove that if there exists a positive integer m for which $a_m = 0$ then either $a_1 = 0$ or $a_2 = 0$.