

# Putnam $\Sigma.4$

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## 1 Problems

**Putnam 2002/A4.** In Determinant Tic-Tac-Toe, Player 1 enters a 1 in an empty  $3 \times 3$  matrix. Player 0 counters with a 0 in a vacant position, and play continues in turn until the  $3 \times 3$  matrix is completed with five 1's and four 0's. Player 0 wins if the determinant is 0 and player 1 wins otherwise. Assuming both players pursue optimal strategies, who will win and how?

**Putnam 2002/A5.** Define a sequence by  $a_0 = 1$ , together with the rules  $a_{2n+1} = a_n$  and  $a_{2n+2} = a_n + a_{n+1}$  for each integer  $n \geq 0$ . Prove that every positive rational number appears in the set

$$\left\{ \frac{a_{n-1}}{a_n} : n \geq 1 \right\} = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{2}, \dots \right\}$$

**Putnam 2002/A6.** Fix an integer  $b \geq 2$ . Let  $f(1) = 1$ ,  $f(2) = 2$ , and for each  $n \geq 3$ , define  $f(n) = nf(d)$ , where  $d$  is the number of base- $b$  digits of  $n$ . For which values of  $b$  does

$$\sum_{n=1}^{\infty} \frac{1}{f(n)}$$

converge?