Putnam $\Sigma.3$

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1 Problems

Putnam 2003/B4. Let

$$\begin{aligned} f(z) &= az^4 + bz^3 + cz^2 + dz + e \\ &= a(z - r_1)(z - r_2)(z - r_3)(z - r_4), \end{aligned}$$

where a, b, c, d, e are integers, $a \neq 0$. Show that if $r_1 + r_2$ is a rational number and $r_1 + r_2 \neq r_3 + r_4$, then $r_1 r_2$ is a rational number.

Putnam 2003/B5. Let A, B, C be equidistant points on the circumference of a circle of unit radius centered at O, and let P be any point in the circle's interior. Let a, b, c be the distances from P to A, B, C, respectively. Show that there is a triangle with side lengths a, b, c, and that the area of this triangle depends only on the distance from P to O.

Putnam 2003/B6. Let $f(x)$ be a continuous real-valued function defined on the interval $[0, 1]$. Show that

$$\int_0^1 \int_0^1 |f(x) + f(y)| dx dy \geq \int_0^1 |f(x)| dx \, .$$