Putnam E.06

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1 Problems

Putnam 1992/B1. Let S be a set of n distinct real numbers. Let A_S be the set of numbers that occur as averages of two distinct elements of S. For a given $n \geq 2$, what is the smallest possible number of elements in A_S?

Putnam 1992/B2. For nonnegative integers n and k, define $Q(n,k)$ to be the coefficient of x^k in the expansion of $(1+x+x^2+x^3)^n$. Prove that

$$Q(n,k) = \sum_{j=0}^{k} \binom{n}{j}\binom{n}{k-2j}.$$

Putnam 1992/B3. For any pair (x,y) of real numbers, a sequence $(a_n(x,y))_{n\geq 0}$ is defined as follows:

$$a_0(x,y) = x\,,$$
$$a_{n+1}(x,y) = \frac{(a_n(x,y))^2 + y^2}{2}\,, \quad \text{for } n \geq 0.$$

Find the area of the region $\{(x,y) : (a_n(x,y))_{n\geq 0} \text{ converges}\}$.