

# Putnam E.06

Po-Shen Loh

7 October 2015

## 1 Problems

**Putnam 1992/A1.** Prove that  $f(n) = 1 - n$  is the only integer-valued function defined on the integers that satisfies the following conditions.

- $f(f(n)) = n$ , for all integers  $n$ ;
- $f(f(n + 2) + 2) = n$ , for all integers  $n$ ;
- $f(0) = 1$ .

**Putnam 1992/A2.** Define  $C(\alpha)$  to be the coefficient of  $x^{1992}$  in the power series about  $x = 0$  of  $(1 + x)^\alpha$ . Evaluate

$$\int_0^1 \left( C(-y - 1) \sum_{k=1}^{1992} \frac{1}{y + k} \right) dy.$$

**Putnam 1992/A3.** For a given positive integer  $m$ , find all triples  $(n, x, y)$  of positive integers, with  $n$  relatively prime to  $m$ , which satisfy

$$(x^2 + y^2)^m = (xy)^n.$$