

Putnam E.02

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1 Problems

Putnam 1994/A1. Suppose that a sequence a_1, a_2, a_3, \dots satisfies $0 < a_n \leq a_{2n} + a_{2n+1}$ for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.

Putnam 1994/A2. Let A be the area of the region in the first quadrant bounded by the line $y = \frac{1}{2}x$, the x -axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$. Find the positive number m such that A is equal to the area of the region in the first quadrant bounded by the line $y = mx$, the y -axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$.

Putnam 1994/A3. Show that if the points of an isosceles right triangle of side length 1 are each colored with one of four colors, then there must be two points of the same color which are at least distance $2 - \sqrt{2}$ apart.