

# Putnam $\Sigma.11$

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## 1 Problems

**Putnam 2006/B4.** Let  $Z$  denote the set of points in  $\mathbb{R}^n$  whose coordinates are 0 or 1. (Thus  $Z$  has  $2^n$  elements, which are the vertices of a unit hypercube in  $\mathbb{R}^n$ .) Given a vector subspace  $V$  of  $\mathbb{R}^n$ , let  $Z(V)$  denote the number of members of  $Z$  that lie in  $V$ . Let  $k$  be given,  $0 \leq k \leq n$ . Find the maximum, over all vector subspaces  $V \subseteq \mathbb{R}^n$  of dimension  $k$ , of the number of points in  $V \cap Z$ . [Editorial note: the proposers probably intended to write  $Z(V)$  instead of “the number of points in  $V \cap Z$ ”, but this changes nothing.]

**Putnam 2006/B5.** For each continuous function  $f : [0, 1] \rightarrow \mathbb{R}$ , let  $I(f) = \int_0^1 x^2 f(x) dx$  and  $J(f) = \int_0^1 x (f(x))^2 dx$ . Find the maximum value of  $I(f) - J(f)$  over all such functions  $f$ .

**Putnam 2006/B6.** Let  $k$  be an integer greater than 1. Suppose  $a_0 > 0$ , and define

$$a_{n+1} = a_n + \frac{1}{\sqrt[k]{a_n}}$$

for  $n > 0$ . Evaluate

$$\lim_{n \rightarrow \infty} \frac{a_n^{k+1}}{n^k}.$$