Putnam Σ.8

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1 Problems

Putnam 1982/B4. A set S of k distinct integers n_i is such that $\prod n_i$ divides $\prod(n_i + m)$ for all integers m. Must 1 or -1 belong to S? If all members of S are positive, is S necessarily just $\{1, 2, \ldots, k\}$?

Putnam 1982/B5. Given $x > e^e$, define the sequence a_n as follows: $a_0 = e$, $a_{n+1} = \frac{\ln x}{\ln a_n}$. Prove that the sequence converges. Let the limit be $f(x)$. Prove that f is continuous.

Putnam 1982/B6. Let $A(a, b, c)$ be the area of a triangle with sides a, b, and c. Let $f(a, b, c) = \sqrt{A(a, b, c)}$. Prove that for any two triangles with sides a, b, c and a', b', c' we have $f(a, b, c) + f(a', b', c') \leq f(a + a', b + b', c + c')$. When do we have equality?