Putnam $\Sigma.7$

Po-Shen Loh

5 October 2014

1 Problems

Putnam 1982/A4. Consider the system of differential equations

$$y' = -z^3$$
$$z' = y^3$$

where y and z are functions from $\mathbb{R} \to \mathbb{R}$, and y' denotes the derivative of y with respect to its variable, etc. Suppose that with initial conditions $y(0) = 1, z(0) = 0$, the system of differential equations has a unique solution $y = f(x), z = g(x)$ for all real x. Prove that $f(x)$ and $g(x)$ are both periodic with the same period.

Putnam 1982/A5. Let a, b, c, d be positive integers satisfying $a + c \leq 1982$ and $\frac{a}{b} + \frac{c}{d} < 1$. Prove that $1 - \frac{a}{b} - \frac{c}{d} > \frac{1}{1983^3}$.

Putnam 1982/A6. Let a_i be real numbers such that $\sum_1^\infty a_i = 1$ and $|a_1| > |a_2| > |a_3| > \cdots$. Suppose that f is a bijection from the positive integers to itself, and

$$|f(i) - i||a_i| \to 0$$

as $i \to \infty$. Prove or disprove that $\sum_1^\infty a_{f(i)} = 1$.