Putnam Σ.13

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1 Problems

Putnam 1986/B4. For a positive real number r, let $G(r)$ be the minimum value of $|r - \sqrt{m^2 + 2n^2}|$ for all integers m and n. Prove or disprove the assertion that $\lim_{r\to\infty} G(r)$ exists and equals 0.

Putnam 1986/B5. Let $f(x, y, z) = x^2 + y^2 + z^2 + xyz$. Let $p(x, y, z), q(x, y, z), r(x, y, z)$ be polynomials with real coefficients satisfying

$$f(p(x, y, z), q(x, y, z), r(x, y, z)) = f(x, y, z).$$

Prove or disprove the assertion that the sequence p, q, r consists of some permutation of $\pm x, \pm y, \pm z$, where the number of minus signs is 0 or 2.

Putnam 1986/B6. Suppose A, B, C, D are $n\times n$ matrices with entries in a field F, satisfying the conditions that AB^T and CD^T are symmetric and $AD^T - BC^T = I$. Here I is the $n \times n$ identity matrix, and if M is an $n \times n$ matrix, M^T is its transpose. Prove that $A^T D - C^T B = I$.