Putnam Σ.09

Po-Shen Loh

20 October 2013

1 Problems

Putnam 1987/A4. Let P be a polynomial, with real coefficients, in three variables and F be a function of two variables such that

$$P(ux, uy, uz) = u^2 F(y - x, z - x) \quad \text{for all real } x, y, z, u,$$

and such that $P(1, 0, 0) = 4$, $P(0, 1, 0) = 5$, and $P(0, 0, 1) = 6$. Also let A, B, C be complex numbers with $P(A, B, C) = 0$ and $|B - A| = 10$. Find $|C - A|$.

Putnam 1987/A5. Let

$$\vec{G}(x, y) = \left(\frac{-y}{x^2 + 4y^2}, \frac{x}{x^2 + 4y^2}, 0 \right).$$

Prove or disprove that there is a vector-valued function

$$\vec{F}(x, y, z) = (M(x, y, z), N(x, y, z), P(x, y, z))$$

with the following properties:

(i) M, N, P have continuous partial derivatives for all $(x, y, z) \neq (0, 0, 0)$;

(ii) $\text{Curl}\, \vec{F} = \vec{0}$ for all $(x, y, z) \neq (0, 0, 0)$;

(iii) $\vec{F}(x, y, 0) = \vec{G}(x, y)$.

Putnam 1987/A6. For each positive integer n, let $a(n)$ be the number of zeroes in the base 3 representation of n. For which positive real numbers x does the series

$$\sum_{n=1}^{\infty} \frac{x^{a(n)}}{n^3}$$

converge?