Putnam Σ.08

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1 Problems

Putnam 1988/B4. Prove that if $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive real numbers, then so is $\sum_{n=1}^{\infty} (a_n)^{n/(n+1)}$.

Putnam 1988/B5. For positive integers n, let M_n be the $2n+1$ by $2n+1$ skew-symmetric matrix for which each entry in the first n subdiagonals below the main diagonal is 1 and each of the remaining entries below the main diagonal is -1. Find, with proof, the rank of M_n. (According to one definition, the rank of a matrix is the largest k such that there is a $k \times k$ submatrix with nonzero determinant.)

One may note that

$$M_1 = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$
$$M_2 = \begin{pmatrix} 0 & -1 & -1 & 1 & 1 \\ 1 & 0 & -1 & -1 & 1 \\ 1 & 1 & 0 & -1 & -1 \\ -1 & 1 & 1 & 0 & -1 \\ -1 & -1 & 1 & 1 & 0 \end{pmatrix}.$$

Putnam 1988/B6. Prove that there exist an infinite number of ordered pairs (a, b) of integers such that for every positive integer t, the number $at + b$ is a triangular number if and only if t is a triangular number. (The triangular numbers are the $t_n = n(n+1)/2$ with n in $\{0, 1, 2, \dots\}$.)