

# Putnam $\Sigma.07$

Po-Shen Loh

6 October 2013

## 1 Problems

**Putnam 1988/A4.** (a) If every point of the plane is painted one of three colors, do there necessarily exist two points of the same color exactly one inch apart?

(b) What if “three” is replaced by “nine”?

**Putnam 1988/A5.** Prove that there exists a *unique* function  $f$  from the set  $\mathbb{R}^+$  of positive real numbers to  $\mathbb{R}^+$  such that

$$f(f(x)) = 6x - f(x)$$

and

$$f(x) > 0$$

for all  $x > 0$ .

**Putnam 1988/A6.** If a linear transformation  $A$  on an  $n$ -dimensional vector space has  $n + 1$  eigenvectors such that any  $n$  of them are linearly independent, does it follow that  $A$  is a scalar multiple of the identity? Prove your answer.