# Putnam Σ.04

Po-Shen Loh

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## 1 Problems

**Putnam 1989/A4.** If $\alpha$ is an irrational number, $0 < \alpha < 1$, is there a finite game with an honest coin such that the probability of one player winning the game is $\alpha$? (An honest coin is one for which the probability of heads and the probability of tails are both $\frac{1}{2}$. A game is finite if with probability 1 it must end in a finite number of moves.)

**Putnam 1989/A5.** Let $m$ be a positive integer and let $\mathcal{G}$ be a regular $(2m+1)$-gon inscribed in the unit circle. Show that there is a positive constant $A$, independent of $m$, with the following property. For any point $p$ inside $\mathcal{G}$ there are two distinct vertices $v_1$ and $v_2$ of $\mathcal{G}$ such that

$$\left|\,|p - v_1| - |p - v_2|\,\right| < \frac{1}{m} - \frac{A}{m^3}.$$

Here $|s - t|$ denotes the distance between the points $s$ and $t$.

**Putnam 1989/A6.** Let $\alpha = 1 + a_1 x + a_2 x^2 + \cdots$ be a formal power series with coefficients in the field of two elements. Let

$$a_n = \begin{cases} 1 & \text{if every block of zeros in the binary expansion of} \\ & n \text{ has an even number of zeros in the block} \\ 0 & \text{otherwise.} \end{cases}$$

(For example, $a_{36} = 1$ because $36 = 100100_2$ and $a_{20} = 0$ because $20 = 10100_2$.) Prove that $\alpha^3 + x\alpha + 1 = 0$.