Putnam E.15

Po-Shen Loh

4 Dec 2013

1 Problems

Putnam 2005/A1. Show that every positive integer is the sum of one or more numbers of the form $2^r 3^s$, where r and s are nonnegative integers and no summand divides another. (For example, $23 = 9+8+6$.)

Putnam 2005/A2. Let $S = \{(a, b) : a = 1, 2, \ldots, n; b = 1, 2, 3\}$. A *rook tour* of S is a polygonal path made up of line segments connecting points p_1, p_2, $\ldots$, p_{3n} in sequence such that

(i) $p_i \in S$,

(ii) p_i and p_{i+1} are a unit distance apart, for $1 \le i < 3n$,

(iii) for each $p \in S$, there is a unique i such that $p_i = p$.

How many rook tours are there that begin at $(1, 1)$ and end at $(n, 1)$?

Putnam 2005/A3. Let $p(z)$ be a polynomial of degree n, all of whose zeros have absolute value 1 in the complex plane. Put $g(z) = p(z)/z^{n/2}$. Show that all zeros of $g'(z) = 0$ have absolute value 1.