

# Putnam E.12

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## 1 Problems

**Putnam 2006/A1.** Find the volume of the region of points  $(x, y, z)$  such that

$$(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2).$$

**Putnam 2006/A2.** Alice and Bob play a game in which they take turns removing stones from a heap that initially has  $n$  stones. The number of stones removed at each turn must be one less than a prime number. The winner is the player who takes the last stone. Alice plays first. Prove that there are infinitely many  $n$  such that Bob has a winning strategy. (For example, if  $n = 17$ , then Alice might take 6 leaving 11; then Bob might take 1 leaving 10; then Alice can take the remaining stones to win.)

**Putnam 2006/A3.** Let  $1, 2, 3, \dots, 2005, 2006, 2007, 2009, 2012, 2016, \dots$  be a sequence defined by  $x_k = k$  for  $k = 1, 2, \dots, 2006$  and  $x_{k+1} = x_k + x_{k-2005}$  for  $k \geq 2006$ . Show that the sequence has 2005 consecutive terms each divisible by 2006.