Putnam E.07

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1 Problems

Putnam 1978/B2. Find

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{i^2j+2ij+ij^2}\,.$$

Putnam 1978/B3. Let the polynomials $p_n(x)$ be defined by

$$p_1(x) = 1 + x$$
$$p_2(x) = 1 + 2x$$
$$p_{2n+1}(x) = p_{2n}(x) + (n+1)x \cdot p_{2n-1}(x)$$
$$p_{2n+2}(x) = p_{2n+1}(x) + (n+1)x \cdot p_{2n}(x)\,.$$

Let a_n be the largest real root of $p_n(x)$. Prove that a_n is monotonic increasing and tends to zero.

Putnam 1978/B4. Show that we can find integers a, b, c, d such that

$$a^2 + b^2 + c^2 + d^2 = abc + abd + acd + bcd\,,$$

and the smallest of a, b, c, d is arbitrarily large.