Putnam E.06

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1 Problems

Putnam 1978/A1. Show that every 20-element subset of $\{1, 4, 7, 10, 13, 16, \ldots, 100\}$ contains two distinct elements which sum to 104.

Putnam 1978/A2. Let A be the real $n \times n$ matrix with diagonal entries $c_1, c_2, \ldots, c_n$, all entries above the diagonal equal to a, and all entries below the diagonal equal to b, which is not equal to a. Prove that

$$\det A = \frac{bp(a) - ap(b)}{b - a},$$

where $p(x) = \prod_{i=1}^{n}(c_i - x)$.

Putnam 1978/A3. Let $p(x) = 2(x^6 + 1) + 4(x^5 + x) + 3(x^4 + x^2) + 5x^3$. Which of these is the smallest?

$$\int_0^\infty \frac{x}{p(x)}dx \qquad \int_0^\infty \frac{x^2}{p(x)}dx \qquad \int_0^\infty \frac{x^3}{p(x)}dx \qquad \int_0^\infty \frac{x^4}{p(x)}dx$$