Putnam E.04

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17 Sep 2013

1 Problems

Putnam 1979/B1. Does there exist a line which is perpendicular to both the curves $y = \sinh x$ and $y = \cosh x$? Recall that

$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2}.$$

Putnam 1979/B2. Let $0 < \alpha < \beta$ be real parameters. Calculate

$$\lim_{\lambda \to 0} \left(\int_0^1 (\beta x + \alpha(1 - x))^\lambda \, dx \right)^{1/\lambda}.$$

Putnam 1979/B3. Let $\mathbb{F}$ be a finite field with n elements, where n is odd, and suppose that $x^2 + bx + c$ is an irreducible polynomial over $\mathbb{F}$. For how many elements $d \in \mathbb{F}$ is $x^2 + bx + d$ irreducible?