Putnam E.02

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1 Problems

Putnam 1980/B1. Determine all real K for which

$$\cosh x \leq e^{Kx^2}$$

holds for all real x. Recall that

$$\cosh x = \frac{e^x + e^{-x}}{2}\,.$$

Putnam 1980/B2. Let S be the region of space defined by the system

$$\begin{aligned} x &\geq 0\,,\\ y &\geq 0\,,\\ z &\geq 0\,,\\ x+y+z &\leq 11\,,\\ 2x+4y+3z &\leq 36\,,\\ 2x+3z &\leq 24\,. \end{aligned}$$

Find the number of vertices and edges of S. For which a, b is

$$ax + by + z \leq 2a + 5b + 4$$

for all points of S?

Putnam 1980/B3. Define a_n by $a_0 = \alpha$, $a_{n+1} = 2a_n - n^2$. For which α are all a_n positive?