Putnam E.01

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28 Aug 2013

1 Problems

Putnam 1980/A1. Let $f(x) = x^2 + bx + c$. Let C be the curve $y = f(x)$ and let P_i be the point $(i, f(i))$ on C. Let A_i be the point of intersection of the tangents at P_i and P_{i+1}. Find the polynomial of smallest degree passing through $A_1, A_2, \ldots, A_9$.

Putnam 1980/A2. In terms of m and n, find the number of 4-tuples (a, b, c, d) of positive integers such that the lowest common multiple of any three integers in the 4-tuple is $3^m 7^n$.

Putnam 1980/A3. Find

$$\int_0^{\pi/2} \frac{dx}{1 + (\tan x)^{\sqrt{2}}} .$$