Putnam D.9

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1 Problems

Putnam 2000/A4. Show that the improper integral

$$\lim_{B \to \infty} \int_0^B \sin(x) \sin(x^2)\, dx$$

converges.

Putnam 2000/A5. Three distinct points with integer coordinates lie in the plane on a circle of radius $r > 0$. Show that two of these points are separated by a distance of at least $r^{1/3}$.

Putnam 2000/A6. Let $f(x)$ be a polynomial with integer coefficients. Define a sequence $a_0, a_1, \ldots$ of integers such that $a_0 = 0$ and $a_{n+1} = f(a_n)$ for all $n \geq 0$. Prove that if there exists a positive integer m for which $a_m = 0$ then either $a_1 = 0$ or $a_2 = 0$.