Putnam C.6

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1 Problems

Putnam 1992/B1. Let S be a set of n distinct real numbers. Let A_S be the set of numbers that occur as averages of two distinct elements of S. For a given $n \geq 2$, what is the smallest possible number of elements in A_S?

Putnam 1992/B2. For nonnegative integers n and k, define $Q(n, k)$ to be the coefficient of x^k in the expansion of $(1 + x + x^2 + x^3)^n$. Prove that

$$Q(n, k) = \sum_{j=0}^{k} \binom{n}{j} \binom{n}{k - 2j} .$$

Putnam 1992/B3. For any pair (x, y) of real numbers, a sequence $(a_n(x, y))_{n \geq 0}$ is defined as follows:

$$\begin{aligned} a_0(x, y) &= x , \\ a_{n+1}(x, y) &= \frac{(a_n(x, y))^2 + y^2}{2} , \quad \text{for } n \geq 0. \end{aligned}$$

Find the area of the region $\{(x, y) : (a_n(x, y))_{n \geq 0} \text{ converges}\}$.