Putnam C.5

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1 Problems

Putnam 1992/A1. Prove that $f(n) = 1 - n$ is the only integer-valued function defined on the integers that satisfies the following conditions.

- $f(f(n)) = n$, for all integers n;
- $f(f(n+2)+2) = n$, for all integers n;
- $f(0) = 1$.

Putnam 1992/A2. Define $C(\alpha)$ to be the coefficient of x^{1992} in the power series about $x = 0$ of $(1+x)^\alpha$. Evaluate

$$\int_0^1 \left(C(-y-1) \sum_{k=1}^{1992} \frac{1}{y+k} \right) dy\,.$$

Putnam 1992/A3. For a given positive integer m, find all triples (n, x, y) of positive integers, with n relatively prime to m, which satisfy

$$(x^2 + y^2)^m = (xy)^n\,.$$