Putnam C.3

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1 Problems

Putnam 1993/A1. The horizontal line $y = c$ intersects the curve $y = 2x - 3x^3$ twice in the first quadrant. Let A be the region bounded by the y-axis, the line $y = c$, and the curve. Let B be the region between the line $y = c$ and the curve, above the segment of $y = c$ that goes between the two intersection points mentioned above. Find c so that the area of region A equals the area of region B.

Putnam 1993/A2. Let $(x_n)_{n\geq 0}$ be a sequence of nonzero real numbers such that

$$x_n^2 - x_{n-1}x_{n+1} = 1 \qquad \text{for } n = 1, 2, 3, \ldots$$

Prove that there exists a real number a such that $x_{n+1} = ax_n - x_{n-1}$ for all $n \geq 1$.

Putnam 1993/A3. Let $\mathcal{P}_n$ be the set of subsets of $\{1, 2, \ldots, n\}$. Let $c(n, m)$ be the number of functions $f : \mathcal{P}_n \to \{1, 2, \ldots, m\}$ such that $f(A \cap B) = \min\{f(A), f(B)\}$. Prove that

$$c(n, m) = \sum_{j=1}^{m} j^n \, .$$