# 13.

(Just do it)

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## 1 Problems

**VTRMC 2003/1.** An investor buys stock worth \$10,000 and holds it for $n$ business days. Each day he has an equal chance of either gaining 20% or losing 10%. However in the case he gains every day (i.e. $n$ gains of 20%), he is deemed to have lost all his money, because he must have been involved with insider trading. Find a (simple) formula, with proof, of the amount of money he will have on average at the end of the $n$ days.

**VTRMC 2002/2.** Find rational numbers $a, b, c, d, e$ such that

$$\sqrt{7+\sqrt{40}} = a + b\sqrt{2} + c\sqrt{5} + d\sqrt{7} + e\sqrt{10}\,.$$

**VTRMC 2003/2.** For $|x| < 1$, find

$$\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)} = \frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots$$

**VTRMC 2005/2.** Find, and write out explicitly, a permutation $(p(1), p(2), ..., p(20))$ of $(1, 2, \ldots, 20)$ such that $k + p(k)$ is a power of 2 for $k = 1, 2, \ldots, 20$, and prove that only one such permutation exists. (To illustrate, a permutation of $(1, 2, 3, 4, 5)$ such that $k + p(k)$ is a power of 2 for $k = 1, 2, \ldots, 5$ is clearly $(1, 2, 5, 4, 3)$, because $1 + 1 = 2$, $2 + 2 = 4$, $3 + 5 = 8$, $4 + 4 = 8$, and $5 + 3 = 8$.)

**IMO 2003/1.** Let $A$ be a subset of the set $S = \{1, 2, \ldots, 1000000\}$ containing exactly 101 elements. Prove that there exist numbers $t_1, t_2, \ldots, t_{100}$ in $S$ such that the sets

$$A_j = \{x + t_j : x \in A\} \quad \text{for } j = 1, 2, \ldots, 100$$

are pairwise disjoint.