5. Calculus

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1 Classical results

Warm-up. Determine $f'(z)$, if

$$f(z) = \int_0^{z^2} e^{-x^2} dx\,.$$

Gaussian. Calculate

$$\int_{-\infty}^{\infty} e^{-x^2} dx\,.$$

2 Problems

VTRMC 2004/5. Let $f(x) = \int_0^x \sin(t^2 - t + x)dt$. Compute $f''(x) + f(x)$, and deduce that $f^{(12)}(0) + f^{(10)}(0) = 0$. (Here, $f^{(10)}$ indicates the 10th derivative.)

VTRMC 2001/1. Three infinitely long circular cylinders, each with unit radius, have their axes along the x, y and z-axes. Determine the volume of the region common to all three cylinders. (Thus one needs the volume common to $\{y^2 + z^2 \leq 1\}$, $\{z^2 + x^2 \leq 1\}$, and $\{x^2 + y^2 \leq 1\}$.)

Putnam 2005/A5. Evaluate

$$\int_0^1 \frac{\ln(x+1)}{x^2+1} dx.$$

VTRMC 2000/3. Consider the initial value problem $y' = y^2 - t^2$; $y(0) = 0$ (where $y' = dy/dt$). Prove that $\lim_{t\to\infty} y'(t)$ exists, and determine its value.