1, Diestel 3.5: Deduce the $k = 2$ case of Menger’s theorem (3.3.1) from Proposition 3.1.1.

2, Diestel 3.17 (i): Find the error in the following ‘simple proof’ of Menger’s theorem (3.3.1). Let $X$ be an $A$–$B$ separator of minimum size. Denote by $G_A$ the subgraph of $G$ induced by $X$ and all the components of $G - X$ that meet $A$, and define $G_B$ correspondingly. By the minimality of $X$, there can be no $A$–$X$ separator in $G_A$ with fewer than $|X|$ vertices, so $G_A$ contains $k$ disjoint $A$–$X$ paths by induction. Similarly, $G_B$ contains $k$ disjoint $X$–$B$ paths. Together, all these paths form the desired $A$–$B$ paths in $G$.

3, Diestel 3.18: Prove Menger’s theorem by induction on $||G||$, as follows. Given an edge $e = xy$, consider a smallest $A$–$B$ separator $S$ in $G - e$. Show that the induction hypothesis implies a solution for $G$ unless $S \cup \{x\}$ and $S \cup \{y\}$ are smallest $A$–$B$ separators in $G$. Then show that if choosing neither of these separators as $X$ in the previous exercise gives a valid proof, there is only one easy case left to do.

4, Diestel 3.21: Let $k \geq 2$. Show that every $k$-connected graph of order at least $2k$ contains a cycle of length at least $2k$.

5, Diestel 3.22: Let $k \geq 2$. Show that in a $k$-connected graph any $k$ vertices lie on a common cycle.