# SOLUTIONS TO HW 3 WITH COMMENTS

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**Everyone did this one quite well. The only question is can I make my solutions even terser than Avi's?**

**I.10** 1 implies 2: the unique prime ideal equal the unique maximal ideal equals the nilradical, all nonunits are in some maximal ideal so all nonunits are nilpotent. 2 implies 3: $a$ not in $\mathfrak{N}$ implies $a$ invertible in $A$ implies $a + \mathfrak{N}$ invertible in $\mathfrak{N}$. 3 implies 1: $\mathfrak{N}$ is maximal, and since it is contained in every prime it is equal to every prime.

**I.14** $\Sigma$ is closed under unions of chains, so has maximal elements by Zorn. Let $P$ be maximal in $\Sigma$. The collection of non zero divisors is closed under multiplication (easy!) so by the usual argument $P$ is prime. Finally if $a$ is a zero divisor then $(a) \in \Sigma$, so $(a) \subseteq P$ for some prime $P$.

**VII.2** We saw in a previous ex that if $f$ is nilpotent then so is each $a_i$. Conversely let all $a_i$ be nilpotent, and let $b_1, \dots b_n$ be a finite set generating the same ideal as the set of $a_i$. Easily $f$ is an $A[[x]]$-linear combination of the $b_j$ and so is nilpotent.