# Homework 4: 21-355–Principles of Real Analysis I

DUE: Friday, October 14, 2016

Name: ______________________________

**Instructions:** Complete the following problems, clearly labeling the problems. Staple this sheet, with your name filled in, to the top of your work. Failure to attach this sheet will result in a five-point deduction in the grade. The assignment will be graded out of one hundred points.

1. Exercise 3.2.2
2. Exercise 3.2.4. Make sure to give a counterexample or proof for your answer to part (b).
3. Exercise 3.2.8
4. Exercise 3.2.15
5. Exercise 3.3.1
6. Exercise 3.3.2
7. Give an example of a bounded, infinite, closed set in $\mathbb{R}$ containing no rational number, or show that such a set cannot exist.
8. Prove or disprove that an open set in $\mathbb{R}$ has no isolated points.
9. Consider a non-empty set $X$ and the function $d : X \times X \mapsto \mathbb{R}$ where

$$d(x, y) = \begin{cases} 1, & \text{if } x \neq y \\ 0, & \text{if } x = y. \end{cases}$$

   This metric space is called a discrete metric space.

   (a) Show that $X$ with the metric $d$ is a metric space.

(b) Show that any subset of $X$ is open under this metric. (This also means from discussion in class that every subset is also closed under this metric.)

10. Let $A = [0, 1)$. Consider the metric space with the set $A$ with the absolute value metric (you may assume without proof that this is a metric space). Show that $\left[0, \frac{1}{2}\right)$ is open in $A$. Conclude that $\left[\frac{1}{2}, 1\right)$ is closed in $A$.