Homework 3: 21-355–Principles of Real Analysis I

DUE: Friday, September 30, 2016

Name: ____

Instructions: Complete the following problems, clearly labeling the problems. Staple this sheet, with your name filled in, to the top of your work. Failure to attach this sheet will result in a five-point deduction in the grade. The assignment will be graded out of one hundred points.

1. Exercise 2.2.2
2. Exercise 2.2.4
3. Exercise 2.3.4 (You may assume that $a_n \neq 0$ for all $n \in \mathbb{N}$. You must explain what algebraic properties of limits you are using to receive credit.)
4. Exercise 2.3.5
5. Exercise 2.3.8
6. Exercise 2.3.12
7. Exercise 2.6.2
8. Exercise 2.6.3
9. Let (a_n) be a real sequence. Show that $\lim_{n\to\infty} a_n = a$ if and only if every open interval containing a contains all but finitely many of the a_n's. Show that this statement is not true if we only require that infinitely many a_n's are in each open interval containing a.
10. Let (a_n) be a real sequence such that $a_n \to a$ as $n \to \infty$. Show that $(|a_n|)$ also converges. Prove or disprove that if $(|a_n|)$ converges, then (a_n) converges.