

## Homework 2: 21-355–Principles of Real Analysis I

DUE: Friday, September 16, 2016

Name: \_\_\_\_\_

**Instructions:** Complete the following problems, clearly labeling the problems. Staple this sheet, with your name filled in, to the top of your work. Failure to attach this sheet will result in a five-point deduction in the grade. The assignment will be graded out of one hundred points.

1. Exercise 1.2.2
2. Exercise 1.2.6
3. Exercise 1.3.3
4. Exercise 1.3.6
5. Exercise 1.3.10
6. Exercise 1.4.4
7. Exercise 1.4.6
8. Exercise 2.3.2
9. Show that the set of *irrational* numbers is dense in  $\mathbb{R}$ .
10. A *metric space* is a set  $X$  paired with a function  $d : X \times X \mapsto \mathbb{R}$  such that  $d$  has the following properties.
  - $d(x, y) \geq 0$  for all  $x, y \in X$  and  $d(x, y) = 0$  if and only if  $x = y$
  - $d(x, y) = d(y, x)$  for all  $x, y \in X$
  - $d(x, y) \leq d(x, z) + d(z, y)$  for all  $x, y, z \in X$

Show that  $(\mathbb{R}, d)$  where  $d(x, y) = |x - y|$  is a metric space. (We will see some other metric spaces later in the course when we discuss spaces of functions.)