Homework 1: 21-355–Principles of Real Analysis I

DUE: Friday, September 2, 2016

Name: ____

Instructions: Complete the following problems, clearly labeling the problems. Staple this sheet, with your name filled in, to the top of your work. Failure to attach this sheet will result in a five-point deduction in the grade. The assignment will be graded out of fifty points.

1. Let $f : X \mapsto Y$. Show that f is onto if and only if $f(f^{-1}(B)) = B$ for all $B \subseteq Y$. **Note:** Here, f^{-1} denotes the preimage of f, not the inverse of f, which may not exist.
2. Prove that $[0, 1]$ is uncountable. Use this fact to argue that $\mathbb{R}$ is uncountable. Then, using the fact that $\mathbb{Q}$ is countable, show that the set of irrational numbers is uncountable.
3. Show that if $f : A \mapsto B$ and $g : B \mapsto C$ are injective, then $g \circ f : A \mapsto C$ is also injective. Is it true that if $g \circ f$ is injective that f and g must also be injective?
4. Let $\widehat{A} = \widehat{C}$ and $\widehat{B} = \widehat{D}$. Prove or disprove that $\widehat{A \times B} = \widehat{C \times D}$. For simplicity, you may assume that none of A, B, C, and D is empty.
5. Using induction, prove that for any $N \in \mathbb{N}$,
$$\left(\bigcup_{i=1}^{N} A_i\right)^c = \bigcap_{i=1}^{N} A_i^c.$$
Find a counter example to show that in general,
$$\left(\bigcup_{i=1}^{\infty} A_i\right)^c \neq \bigcap_{i=1}^{\infty} A_i^c.$$