21-241 Lec3 Final Exam Study Guide

If you understand and can do the following things as well as the topics on the previous three study guides, you should do well on the final exam. Remember, it is closed book, closed notes, no electronic devices except internet-less calculators, etc. You are allowed one 8.5 inch by 11 inch sheet of paper with whatever material you wish to put on it. You should also be comfortable writing mathematical proofs. On the exam, your proofs should meet the following specifications:

1. everything stated should be true and explained where appropriate (Yes, I know this is vague, but sometimes this is a stylistic decision. When in doubt, explain why your statement is true.),
2. the proof needs to prove the actual statement in question, and
3. the proof needs to be written in clear mathematical English (that is, there should be words explaining what you are doing).

1 General Vector Spaces

You should be able to do/be comfortable with

- bases of vector spaces
- change of basis matrices.

2 Linear Transformations

You should be able to do and calculate/be comfortable with

- definition of linear transformations between general vector spaces
- proving a mapping is a linear transformation
- definitions and showing a mapping is injective (1-1), surjective (onto), and bijective
- compositions of linear transformations
- inverses of linear transformations

- kernel and range of linear transformations
- rank theorem for linear transformations
- isomorphisms and isomorphic spaces
- matrices of linear transformations
- compositions and inverses of linear transformations and their relationship with the matrices of the linear transformations
- the Fundamental Theorem of Invertible Matrices in Chapter 6 of your book.

3 Inner Product Spaces

You should be able to do and calculate/be comfortable with

- definition of inner product
- properties of inner products
- norm of a vector for a given inner product
- orthogonality for general inner product spaces
- Gram-Schmidt process for general inner product spaces
- Cauchy-Schwarz inequality for general norms
- norms and distance
- matrix norms.

4 Quadratic Forms

You should be able to do and calculate/be comfortable with (we may not get to all of these by the end of the semester)

- definition of quadratic forms
- Principal Axes Theorem
- Singular Value Decomposition