# 21-241 Lec3 Exam 3 Study Guide

If you understand and can do the following things, you should do well on the second exam. Remember, it is closed book, closed notes, no electronic devices except internet-less calculators, etc. You should also be comfortable writing mathematical proofs. On the exam, your proofs should meet the following specifications:

1. everything stated should be true and explained where appropriate (Yes, I know this is vague, but sometimes this is a stylistic decision. When in doubt, explain why your statement is true.),
2. the proof needs to prove the actual statement in question, and
3. the proof needs to be written in clear mathematical English (that is, there should be words explaining what you are doing).

## 1 Matrices

You should know/be able to do

- how to calculate $A^k\mathbf{x}$ where $\mathbf{x}$ is a linear combination of eigenvectors of $A$,
- similarity of matrices,
- what properties similar matrices share,
- diagonalizable matrices and how to diagonlaize them,
- the Diagonalization Theorem.

## 2 Orthogonality

You need to be comfortable with/comfortable with using

- orthgonal and orthonormal sets of vectors in $\mathbb{R}^n$,
- properties of orthogonal vectors,
- orthogonal and orthonormal bases of subspaces of $\mathbb{R}^n$,

- how to calculate the coordinates of a vector in a subspace with respect to an orthogonal basis of that subspace,
- orthogonal matrices and their properties,
- orthogonal projections,
- orthogonal complements
- how the four fundamental spaces of an $m \times n$ matrix are related to each other in regards to orthogonal complements, and using these facts to calculate bases of the orthogonal complement of a subspace of $\mathbb{R}^n$,
- how to construct orthogonal and orthonormal bases of a subspace using Gram-Schmidt,
- how to calculate a $QR$ factorization of a matrix,
- orthogonal diagonalization,
- the spectral theorem and spectral decomposition.

## 3 General Vector Spaces

You need to be comfortable with/comfortable with using

- the definition of a vector space,
- properties of vector spaces proven in class and on the homework,
- subspaces of vector spaces,
- the spaces $\mathcal{F}$, $M^{m,n}$, $\mathbb{R}^n$, $P_n$, $\mathcal{P}$,
- span of vectors in a vector space,
- linear independence/dependence in a vector space,
- bases of vector spaces,
- coordinate vectors,
- dimension of a vector space,
- change of basis of vector spaces.

# 4 Note About Using Calculators

As I have said in class, you are allowed to use graphing calculators, so long as they do not have the capability of connecting to any computer network. As such, it is not expected on the exam that you do row reductions by hand or calculate inverses of matrices by hand (granted, this second one is a case of row reduction). When you are showing your work on the exam, it is sufficient to make clear which matrix you are putting into row echelon form or reduced row echelon form (indicating which one of the two), and then giving the reduced form, labeling it as such. For example, if you need to find the reduced row echelon form of the matrix $A$, for whatever reason, you may write

$$\mathrm{rref}(A) = \cdots$$

where $\cdots$ is the reduced row echelon form of the matrix. When calculating the determinant of a matrix, you should indicate the cofactor expansion you are using to reduce the problem to finding determinants of $2 \times 2$ matrices.