Answer the following questions on the given sheets of paper. Make sure that each sheet corresponds to only ONE problem. You may use the backs of the sheets of paper if you wish. You must show your work to receive full credit. Make sure your answers are legible and clear. No notes, crib sheets, calculators, books, phones, computers, or other electronic devices are allowed. You have 50 minutes for the exam. Staple this sheet to the top of your work when you turn in the exam, and then place your exam on the pile for your section.

Name: ______________________________

Section: ______________________________

1. (a) (10 points) Fix $\mathbf{u}$ in $\mathbb{R}^n$ such that $\mathbf{u} \neq \mathbf{0}$. Prove that for any $\mathbf{v} \in \mathbb{R}^n$,

$$\|\mathbf{proj_u v}\| \leq \|\mathbf{v}\|.$$

   (b) (15 points) Let l be the line given by the equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

   and m be the line given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}.$$

   Determine if l and m intersect, and if so, where.

2. (a) (10 points) Show that the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

   spans $\mathbb{R}^2$.

   (b) (10 points) Show that

$$\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

   are linearly dependent.

   (c) (5 points) Give the system of linear equations and the augmented matrix for the system used to balance the chemical equation

$$C_7H_6O_2 + O_2 \to H_2O + CO_2.$$

   You do not need to solve the system. Make sure to label what each unknown represents.

3. (a) (10 points) After a finite number of elementary row operations, the augmented matrix of a system of linear equations in the unknowns x, y, z, and t is put into the following row echelon form:

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & 5 & -6 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Put this augmented matrix into *reduced* row echelon form and solve the system.

(b) Consider the linear system in the unknowns x and y given by

$$x + 3y = k$$

$$4x + hy = 8.$$

i. (5 points) Determine for which values of h and k the system has no solutions.

ii. (5 points) Determine for which values of h and k has infinitely many solutions and express them in terms of h and k.

iii. (5 points) Determine for which values of h and k the system has a unique solution and give it in terms of h and k.

4. (a) (15 points) Let

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}.$$

Calculate AA^T.

(b) (10 points) Let

$$B = \begin{bmatrix} 1 & 0 & 4 \\ -2 & 7 & 0 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 5 & 8 & 42 \\ -9 & 0 & 1 \end{bmatrix}.$$

Calculate $B + C$ and explain why BC is undefined.