Homework 5–21-241 Lec3, Matrices and Linear Transformations

Name: _____
Section: _____

Instructions: Complete the following problems, clearly labeling the problems. Staple this sheet, with your name and section filled in, to the top of your work. Failure to attach this sheet will result in a one point deduction in the grade. The assignment will be graded out of fifty points.

DUE: Monday, March 6, 2017

Book Problems

- Section 4.1: 6, 12
- Section 4.2: 10, 14, 26, 36, 46, 52, 56
- Section 4.3: 8, 16, 24

Other Problem

1. For a function $f : A \mapsto B$ for some sets A and B, f is onto, or surjective, if and only if for each $b \in B$, there is some $a \in A$ such that $f(a) = b$. We call f one-to-one, or injective, if and only if for any $x, y \in A$ such that $f(x) = f(y)$, we have that $x = y$.

   Let $T : \mathbb{R}^n \mapsto \mathbb{R}^n$ be a linear transformation with standard matrix A, that is, $T(\mathbf{x}) = A\mathbf{x}$ for each $\mathbf{x} \in \mathbb{R}^n$. Show that A is invertible if and only if T is one-to-one and onto.

2. We will look at some of the pitfalls of using computers to calculate determinants

(a) Using MATLAB, calculate the determinant of

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

You should get that the determinant is zero (you can verify this by hand if you wish).

(b) Show, by hand, that for any $\varepsilon > 0$,

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9+\varepsilon \end{bmatrix}\right) = -3\varepsilon.$$

This means the matrix in this part is invertible no matter what $\varepsilon > 0$ is used.

(c) Use MATLAB to calculate the determinant of the matrix in the previous part for $\varepsilon = 10^k$ for various positive integers k. What happens to the calculation?